**Academic Year 2023-2024**

**Exam 1 – Overall assessment - Maximum duration: 3 hours**

**Problem 1 [3.5 points]**

The dynamics of a non-linear electronic circuit are given by the following differential equation:

where is the input voltage, is the output voltage, and is a parameter (constant) whose value determines the behaviour of the circuit.

* 1. Analyse the possible equilibria of the system and calculate the corresponding valid internal linear descriptions (matrices and ) around the equilibria (operating point) given by , as well as the transfer functions of the system valid around that operating point. **[1 point]**
  2. For and using the transfer functions (linear models) that describe the behaviour of the system around each of the equilibria, determine the local stability of the system for different values of in the range ( (you can use the Routh-Hurwitz criterion or calculate the roots directly). Draw the locus of the roots of the open-loop transfer function for different values of only in the case of stable equilibrium. **[1 point]**
  3. For stable equilibrium, calculate the mathematical expression and draw qualitatively (indicating only a few representative points) the unit step response of the linear system: and **[0.25 points]** and for the case and **[0.25 points]**.
  4. For stable equilibrium, calculate the mathematical expression and draw a qualitative diagram (indicating only a few representative points) of the free response of the system considering and , starting from an initial value of the linearised state given by **[0.75 points]**
  5. Draw a Simulink diagram that allows you to simulate and compare the non-linear and linear models for the simulation scenario described in section 1.4. Include the values of the input blocks, constants, and integrators. **[0.25 points]**

**Problem 2 [1.5 points]**

For the dynamic systems described by the following transfer functions, draw qualitatively (indicating the most representative points) the indicated open-loop time response, the Bode diagram, and the Nyquist diagram of .

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| --- | --- | --- | --- |
| **System** | **Unit step response** | **Bode** | **Nyquist** |
|  | **[0.25]** | **[0.25]** | **[0.25]** |
| **System** | **Unit step response** | **Bode** | **Nyquist** |
|  | **[0.25]** | **[0.25]** | **[0.25]** |

**Problem 3 [5 points]**

A dynamic system is described by the following transfer function:

1. Calculate an internal description (state space) of the system represented by Assuming that there are no sensors available to measure the states of (only the values of the system input and output are known), design a control system that rejects disturbances at the system input (the reference is zero), imposing a closed-loop dynamics characterised by equal real poles at and an observation error dynamics characterised by real poles equal to. Verify that the system is controllable and observable and draw the complete block diagram including the control and state observer with the highest possible degree of definition (using integrator blocks to represent the relationship between the states and their derivatives). You should also indicate the equations that provide the evolution of the estimated states and the control signal (write the equation for each estimated state). When calculating the control gain vector H and the observer vector L, leave all the calculations as indicated, but it is not necessary to perform the inverse of the matrices involved. **[1.5 points]**
2. Reasoning using the root locus technique:
   1. raw the root locus when the system represented by is controlled by a proportional controller. Are there values of the controller gain that allow the system to be stabilised (at least marginally) in closed loop? If so, indicate the range of values. **[0.25 points]**
   2. Draw the corresponding root locus and design the simplest possible controller that achieves a first-order closed-loop system with a time constant of 10 seconds. What is the steady-state error when the reference is a unit step?  **[0.5 points]**
   3. Draw the corresponding root locus and design a controller that achieves a steady-state error of zero for a step input to the reference and a closed loop with two equal poles at . Indicate the controller gain value at which the closed-loop system is stable. **[1 point]**
3. Analyse the stability of the closed-loop system in section 2.c (including the controller and leaving its gain free) using Nyquist's stability criterion and indicating the values of with which a stable, marginally stable and unstable closed loop is achieved. **[1 point]**
4. Design a phase lag or phase lead network (a simple analysis of the root locus can help choose the type of network) to control the system represented by such that the closed loop has a steady-state error at step input at the reference equal to and a phase margin greater than or equal to 60º. Does it make sense to use phase margin as a specification? What is the gain margin? **[0.75 points]**